PROCEEDINGS

1st International Congress of Serbian Society of Mechanics

Editors
Dragoslav Šumarac and Dragoslav Kuzmanović

10-13th April, 2007, Kopaonik

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PREFACE

This proceedings contains the papers presented at the First International Congress of Serbian Society of Mechanics held in Kopaonik during the period 10^{th} – 13^{th} April, 2007. This Congress is successor of series of YU Congress and it represents 26^{th} YU Congress of Mechanics. Once again, the high standard of the submitted papers, of which there were over two hundred, made the selection of the contributed papers an extremely difficult task for the organizing committee. Regrettably, many papers had to be rejected on the basis of insufficient space and time but the organizers would like to thank the authors concerned for sending in their manuscript for consideration.

The continuing rapid escalation of all fields in Mechanics are demonstrated in the current congress proceedings. The papers, contributed by authors from all around the globe, have been separated into 10 sections which cover the main areas of interest, e.g. 'Plenary lectures', 'Section A', Section B', Section C', Section D' and Five Minisiposia.

In the Proceedings we incorporated 'A Tribute to Milutin Milanković', great Serbian scientist in the field of Mechanics and Astrophysics. Regrettably so far there was no presence of Prof. Milanković at previous Congresses, even he was alive when YU Society of Mechanics was established in 1954.

The proceedings are printed from direct submitted files of authors and the editors do not accept responsibility for any erroneous comments or opinions expressed herein.

Last, but by no means least, the Congress organizing committee wishes to acknowledge the collaboration of the Ministry of Science and environmental protection, Serbian Chamber of Engineers, Municipality of Raška and Many Supporting members of the Serbian Society of Mechanics listed in the proceedings.

D. ŠUMARAC & D. KUZMANOVIĆ
March, 2007.

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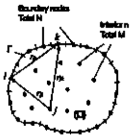

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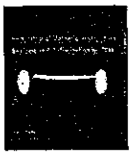

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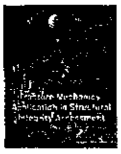

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THE PENALTY METHODS APPLIED TO NONLINEAR CONTACT PROBLEMS

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Abstract:

Approved finite element analysis programs use contact discretizations based on the so-called node-to-segment (NTS) element. In this paper finite node-to-segment contact element, based on the penalty method, is developed. The presented approach, based on a non-associated frictional law, elastic-plastic tangential slip decomposition, and consistent symbolic linearization, results in quadratic rates of convergence within the Newton-Raphson iteration. Standard procedures are used for the detection of contact and for the application of displacement constraints.

The developed algorithm has been implemented into the version of the computational finite element PAK program. Example demonstrate the effectiveness of using the presented approach.

Key words: contact problem, friction, penalty method

1. Introduction

Many physical systems require the description of mechanical interaction across interface if they are to be successfully analyzed. Examples in the engineering word range from description of the interaction between soil and foundations in civil engineering; to design of prosthetics in biomedical engineering; to development of pneumatic tires with better handling characteristics in automotive engineering. The development of more efficient, fast and stabile finite element contact discretizations is still a hot topic, especially due to the fact that engineering applications become more and more complex.

The penalty formulation has the advantage that it is purely geometrically based and therefore no additional degrees of freedom must be activated or inactivated. Numerical example is shown to demonstrate that the presented algorithm can be successfully applied to real contact problems.

2. Formulation of the multi-body frictional contact problem

As the configurations of two bodies coming into the contact are not a priori known, contact represents a nonlinear problem even when the continuum behaves as a linear elastic material.

Using a standard notation in contact mechanics, for each pair of contact surfaces, involved in the problem, we will define slave ($\Gamma_C^{(1)}$) and master surfaces ($\Gamma_C^{(2)}$), Fig. 1. The condition which must be satisfied is that any slave particle cannot penetrate the master surface.

Let $\bar{\mathbf{x}}$ be the projection point of the current position of the slave node $\mathbf{x}^s$ onto current position of the master surface $\Gamma_C^{(2)}$, defined as

$$\frac{\mathbf{x}^k - \bar{\mathbf{x}}(\bar{\xi}^1,\bar{\xi}^2)}{\left\|\mathbf{x}^k - \bar{\mathbf{x}}(\bar{\xi}^1,\bar{\xi}^2)\right\|} \cdot \bar{\mathbf{a}}_\alpha(\bar{\xi}^1,\bar{\xi}^2) = 0, \tag{1}$$

where $\alpha = 1,2$ and $\bar{\mathbf{a}}_\alpha(\bar{\xi}^1,\bar{\xi}^2)$ are the tangent covariant base vectors at the point $\bar{\mathbf{x}}$. The normal gap or the penetration g_N for slave node k is defined as the distance between current positions of this node to the master surface $\Gamma_C^{(2)}$:

$$g_N = (\mathbf{x}^k - \bar{\mathbf{x}}) \cdot \bar{\mathbf{n}}, \tag{2}$$

where $\bar{\mathbf{n}}$ refers to the normal to the master face $\Gamma_C^{(2)}$ at point $\bar{\mathbf{x}}$ (Fig. 1). Normal will be defined using tangent vectors at the point $\bar{\mathbf{x}}$

$$\bar{\mathbf{n}} = \frac{\bar{\mathbf{a}}_1 \times \bar{\mathbf{a}}_2}{\left\|\bar{\mathbf{a}}_1 \times \bar{\mathbf{a}}_2\right\|}, \tag{3}$$

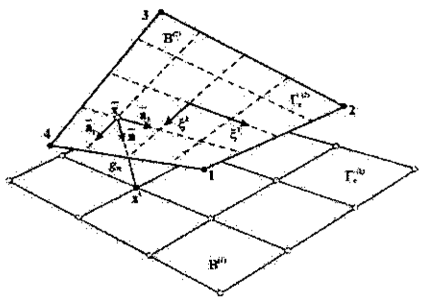

Fig. 1. Geometry of the 3D node-to-segment contact element

This gap (2) gives the non-penetration conditions as follows

$$g_N = 0 \ \text{perfect contact}; \quad g_N > 0 \ \text{no contact}; \quad g_N < 0 \ \text{penetration}, \tag{4}$$

If the analyzed problem is frictionless, function (4) completely defines the contact kinematics. However, if friction is modeled, tangential relative displacement must be introduced. In that case, the sliding path of the node $\mathbf{x}^k$ over the contact surface $\Gamma_C^{(2)}$ is described by total tangential relative displacement as

$$g_T = \int_{t_0}^{t} \left\|\dot{\mathbf{g}}_T\right\| dt = \int_{t_0}^{t} \left\|\dot{\bar{\xi}}^\alpha \bar{\mathbf{a}}_\alpha\right\| dt = \int_{t_0}^{t} \sqrt{\dot{\bar{\xi}}^\alpha \dot{\bar{\xi}}^\beta a_{\alpha\beta}}\, dt, \tag{5}$$

in time interval from t_0 to t.

The time derivatives of parameter $\bar{\xi}^\alpha$ in equation (5) can be computed from (1), [5]. In the geometrically linear case we obtain the following result

$$\bar{a}_{\beta\alpha} \dot{\bar{\xi}}^\beta = \left[\dot{\mathbf{x}}^k - \dot{\bar{\mathbf{x}}}\right] \cdot \bar{\mathbf{a}}_\alpha = \dot{g}_{T\alpha}, \tag{6}$$

where $\bar{a}_{\alpha\beta} = \bar{\mathbf{a}}_\alpha \cdot \bar{\mathbf{a}}_\beta$ is the metric tensor in point $\bar{\mathbf{x}}$ of the master surface $\Gamma_C^{(2)}$. From the equations (5) and (6) we can express the relative tangential velocity at the contact point

$$\dot{\mathbf{g}}_T = \dot{\bar{\xi}}^\alpha \bar{\mathbf{a}}_\alpha = \dot{g}_{T\alpha} \bar{\mathbf{a}}^\alpha, \tag{7}$$

2.1 Constitutive equations for contact interface

A contact stress vector $\bar{\mathbf{t}}$ with respect to the current contact interface $\Gamma_C^{(2)}$ can be split into a normal and tangential part.

$$\bar{\mathbf{t}} = \bar{\mathbf{t}}_N + \bar{\mathbf{t}}_T = \bar{t}_N \bar{\mathbf{n}} + \bar{t}_{T\alpha} \bar{\mathbf{a}}^\alpha , \tag{8}$$

where $\bar{\mathbf{a}}^\alpha$ is contravariant base vector. The stress acts on both surfaces according to the action-reaction principle: $\bar{\mathbf{t}}(\bar{\xi}^1, \bar{\xi}^2) = -\mathbf{t}$ in the contact point $\bar{\mathbf{x}}$. The tangential stress $\bar{t}_{T\alpha}$ is zero in the case of frictionless contact. In the case of contact there is condition $\bar{t}_N < 0$. If there is not penetration between the bodies, then relations $g_N > 0$ and $\bar{t}_N = 0$ hold. This leads to the statements

$$g_N \geq 0, \quad \bar{t}_N \leq 0, \quad \bar{t}_N g_N = 0 , \tag{9}$$

which are known as Kuhn-Tucker conditions. Using the penalty method for normal stress, constitutive equation can be formulated as

$$t_N = \varepsilon_N g_N , \tag{10}$$

where ε_N is the normal penalty parameter.

In tangential direction there is difference between stick and slip. As long as no sliding between two bodies occurs, the tangential relative velocity is zero. If the velocity is zero, also the tangential relative displacement (5) is zero. This state is called stick case with the following restriction:

$$\dot{\mathbf{g}}_T = \mathbf{0} \Leftrightarrow \mathbf{g}_T = \mathbf{0} , \tag{11}$$

For stick a simple linear constitutive model can be used to describe the tangential stress.

$$t_{T\alpha}^{stick} = \varepsilon_T g_{T\alpha} , \tag{12}$$

where ε_T is the tangential penalty parameter.

A relative movement between two bodies occurs if the static friction resistance is overcome and the loading is large enough such that the sliding process can be kept. The tangential stress vector is restricted as follows:

$$t_{T\alpha}^{sl} = -\mu \left| \mathbf{t}_N \right| \frac{\dot{g}_{T\alpha}^{sl}}{\left\| \dot{\mathbf{g}}_T^{sl} \right\|} , \tag{13}$$

where μ is friction coefficient. In the simplest form of Coulomb's law (13), μ is constant so there is no difference between static and sliding friction.

After the introduction of the stick and slip constraints, we need to introduce indicator to define whether stick or slip actually take place. Therefore an indicator function

$$f = \left\| \mathbf{t}_T \right\| - \mu \left| t_N \right| , \tag{14}$$

is evaluated, which respect the Coulomb's model for frictional interface law. In the equation (14) the first term is $\left\| \mathbf{t}_T \right\| = \sqrt{t_{T\alpha} \bar{a}^{\alpha\beta} t_{T\beta}}$.

A backward Euler integration scheme and return mapping strategy are employed to integrate the friction equations (14). If a state of stick is assumed, the trial values of the tangential contact pressure vector $t_{T\alpha}$, and the indicator function f at load step $n+1$ can be expressed in terms of their values at load step n as follows

$$t_{T\alpha\ n+1}^{trial} = t_{T\alpha\ n} + \varepsilon_T \Delta g_{T\alpha\ n+1} = t_{T\alpha\ n} + \varepsilon_T \bar{a}_{\alpha\beta} \Delta\xi_{n+1}^{\beta}, \tag{15}$$

$$f_{Tn+1}^{trial} = \left\| \mathbf{t}_{Tn+1}^{trial} \right\| - \mu \left| t_{Nn+1} \right|, \tag{16}$$

The return mapping is completed by

$$t_{T\alpha\ n+1} = \begin{cases} t_{T\alpha\ n+1}^{trial} & \text{if } f \le 0 \\ \mu \left| t_{Nn+1} \right| n_{T\alpha\ n+1}^{trial} & \text{if } f > 0 \end{cases}, \tag{17}$$

with

$$n_{T\alpha\ n+1}^{trial} = \frac{t_{T\alpha\ n+1}^{trial}}{\left\| \mathbf{t}_{Tn+1}^{trial} \right\|}, \tag{18}$$

For the both cases, the penalty method can be illustrated as a group of linear elastic springs that force the body back to the contact surface when overlapping or sliding occurs.

3. Algorithm for frictional contact

For solution a nonlinear equilibrium equation with inequality constraints (4) as a result of contact, we use a standard implicit method. In order to apply Newton's method for the solution system of equilibrium equation, a linearization of the contact contributions is necessary. In this paper, we do not state the linearization procedure for standard finite element formulation as well as the contact interface law for the normal and tangential part. It could be found in [5].

The tangent stiffness matrix for the normal contact is

$$\mathbf{K}_N = \varepsilon_N \mathbf{N}\mathbf{N}^T \tag{19}$$

Table 1. Frictional contact algorithm using the penalty method

```
LOOP over all contact segment k
 (check for contact (6))  IF g_N ≤ 0 THEN
     (the first iteration)        IF i=1 THEN
                                    set all active nodes to state stick,
                                    t_Tn+1 (18), compute matrix K_T^stick
                                  ELSE
                                    Compute trial state: t^trial_Tα n+1 (19) and f^trial_Tn+1 (20)
                                    IF f^trial_Tn+1 ≤ 0 THEN
                                      t_Tα n+1 = t^trial_Tα n+1 , compute matrix K_T^stick (40)
                                      GO TO (a)
                                    ELSE
                                      t_Tα n+1 = μ|t_Nn+1| n^trial_Tα n+1 , compute matrix K_T^slip (43)
                                    ENDIF
                                  ENDIF
                              ENDIF
(a)  END LOOP
```

The symmetric tangent stiffness matrix for stick condition is

$$\mathbf{K}_T^{stick} = \varepsilon_T \bar{a}_{\alpha\beta} \mathbf{D}^{\alpha} \mathbf{D}^{\beta T} \tag{20}$$

where

$$\mathbf{N}^T = \{\bar{\mathbf{n}} \quad -H_1\bar{\mathbf{n}} \quad -H_2\bar{\mathbf{n}} \quad -H_3\bar{\mathbf{n}} \quad -H_4\bar{\mathbf{n}}\}^T \ ,$$

$$\mathbf{T}_\beta^T = \{\bar{\mathbf{a}}_\beta \quad -H_1\bar{\mathbf{a}}_\beta \quad -H_2\bar{\mathbf{a}}_\beta \quad -H_3\bar{\mathbf{a}}_\beta \quad -H_4\bar{\mathbf{a}}_\beta\}^T, \quad \mathbf{D}^\alpha = \bar{a}^{\alpha\beta}\mathbf{T}_\beta . \tag{21}$$

The linearization of $n_{T\alpha\, n+1}^{trial}$ gives

$$\Delta\left(n_{T\alpha\, n+1}^{trial}\right) = \Delta\left(\frac{t_{T\alpha\, n+1}^{trial}}{\left\|\mathbf{t}_{Tn+1}^{trial}\right\|}\right) = \frac{1}{\left\|\mathbf{t}_{Tn+1}^{trial}\right\|}\left[\delta_\alpha^\beta - n_{T\alpha\, n+1}^{trial} n_{T\, n+1}^{trial\,\beta}\right]\Delta t_{T\beta\, n+1}^{trial} \tag{22}$$

The tangent stiffness matrix for slip condition is

$$\mathbf{K}_T^{slip} = \mu\varepsilon_N n_{T\alpha\, n+1}^{trial}\mathbf{D}^\alpha\mathbf{N}^T + \frac{\mu\varepsilon_N g_{Nn+1}}{\left\|\mathbf{t}_{Tn+1}^{trial}\right\|}\varepsilon_T\bar{a}_{\beta\gamma}\left[\delta_\alpha^\beta - n_{T\alpha\, n+1}^{trial} n_{T\, n+1}^{trial\,\beta}\right]\mathbf{D}^\alpha\mathbf{D}^{\gamma T} \tag{23}$$

The second term, the tangent matrix is non-symmetric.
Frictional contact algorithm using penalty method is shown in Table 1.

4. Example

The roll bar is one very important part of safety equipment in sport car. According to FIA standards, roll bar needs to contain basic arc, auxiliary arc, front arc, and two lateral arcs, several diagonal arcs and few lamellas. The basic arc that stands behind head of driver represents basic part of the structure. In the case of the vehicle rollover it is exposed to the maximal load and displacement. According to FIA standards, the structure has to satisfy quasi-static pressure test. Experiments can be done for basic and auxiliary arcs. In this example, quasi-static test for basic arc of roll bar, which is built in YUGO car, is simulated.

The three node shell element is used for modeling of roll bar. Deformation of the basic arc is increased by prescribed displacement at the rectangular plate. Solution is obtained by 50 steps of displacement increments equal to 1 mm.

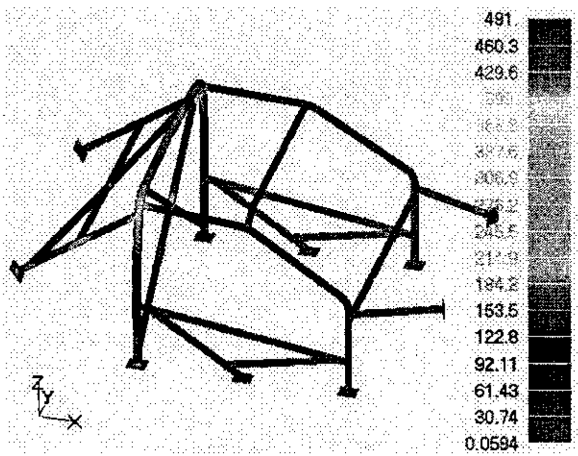

Fig. 2 Effective stress field